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PROVED FIXED POINT RESULT IN A FUZZY INTITIONISTIC METRIC SPACE

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ABSTRACT

Carrier frequency offset is the major problem associated with OFDM systems as it causes the loss of orthogonality of sub-carriers & hence results into Inter Carrier Interference (ICI). So, the minimization of (ICI) is one of the most important challenges for OFDM system developers. Many possible methods exist to minimize/remove ICI upto the maximum extent. The aim of this paper is to focus on the use of Kalman Filter for ICI cancellation in OFDM system. This paper also shows the simulation result under specified parameters & thus investigates its performance in reducing ICI.

KEYWORDS: OFDM, ICI, Kalman Filter.

INTRODUCTION

In mathematics, the fixed-point theorem is a formula that counts the fixed points of a continuous mapping from a compact topological space *X* to itself by means of traces of the induced mappings on the homology groups of *X*. It is named after Solomon Lefschetz, who first stated it in 1926. Counting is subject to an imputed multiplicity at a fixed point called the fixed point index. Weak version of the theorem is enough to show that a mapping without *any* fixed point must have rather special topological properties (like a rotation of a circle). In previous, Liu, Wu and Li defined common (E.A) property in metric spaces and proved common fixed point theorems under strict contractive conditions. Jungck and Rhoades initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly compatible but reverse is not true. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering, and economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc. Recently, Kumar established some common fixed point theorems in intuitionistic fuzzy metricspace using property (E.A) [1-4]. We generalize his result for E.A. Like property, and prove fixed theorems for weakly compatible mappings via an implicit relation in intuitionistic fuzzy metric spaces.

PRELIMINARIES OF FIXED POINT THEOREM AND IINTITIONISTIC FUZZY METRIC SPACE

Definition 2.1: A binary operation $*$: $[0; 1] \times [0; 1] \rightarrow [0; 1]$ is continuous t-norm if satisfies the following conditions: (1) $*$is commutative and associative, (2) $*$is continuous, (3) $a * 1 = a$ for all $a \in [0; 1]$, (4) $a *b \leq c * d$ whenever $a\leq c$ and $b\leq d$ for all $a, b, c, d \in [0; 1]$.

Definition 2.2: A binary operation $\Diamond$: $[0; 1]\times[0; 1]\rightarrow [0; 1]$ is continuous t-norm if satisfies the following conditions: (1) $\Diamond$ is commutative and associative,

(2) $\Diamond$ is continuous,

(3) $a \Diamond 0 = a$ for all $a \in [0; 1]$,

(4) $a \Diamond b \leq c\Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0; 1]$.

The idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzymetric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzymetric space due to Kramosil and Michalek [ ] as:

Definition 2.3: A 5-tuple $(X; M; N; *; \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0; \infty)$ satisfying the following conditions:

(1) $M(x, y, t) + N(x,y, t) \leq 1$ for all $x, y \in X$ and $t > 0$,

(2) $M(x, y, 0) = 0$ for all $x, y \in X$,
(3) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$,
(4) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$,
(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$,
(6) for all $x, y \in X$, $M(x, y,.) : [0, \infty) \to [0, 1]$ is left continuous,
(7) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$,
(8) $N(x, y, 0) = 1$ for all $x, y \in X$,
(9) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$,
(10) $N(x, y, t) = N(y, x, t)$ for all $x; y \in X$ and $t > 0$,
(11) $N(x, y, t) \lozenge N(y, z, s) = N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$,
(12) for all $x, y \in X$, $N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous,
(13) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4: In intuitionistic fuzzy metric space $(X, M, N,*, \lozenge)$, $M(x, y, *)$ is non-decreasing and $N(x, y, \lozenge)$ is non-increasing for all $x, y \in X$.

Definition 2.5: Let $(X, M, N,*, \lozenge)$, be an intuitionistic fuzzy metric space. Then: a sequence $\{xn\}$in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,
$\lim_{n\to\infty} M(xn+p, x, t) = 1$ and $\lim_{n\to\infty} N(xn+p, x, t) = 0$:

Definition 2.6: a sequence $\{xn\}$in X is said to be convergent to a point $x \in X$ if, for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(xn, x, t) = 1$ and $\lim_{n\to\infty} N(xn, x, t) = 0$:) a sequence fxng in X is said to be convergent to a point x 2 X if, for all $t > 0$,

Definition 2.7: An intuitionistic fuzzy metric space $(X, M, N,*, \lozenge)$, is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.8: A pair of self mappings (T, S) of an intuitionistic fuzzy metric space $(X, M, N,*, \lozenge)$, is said to satisfy the property (E.A) if there exist a sequence $\{xn\}$in X such that $\lim_{n\to\infty} Txn= \lim_{n\to\infty} Sxn=z$ in X.

Definition 2.9: A pair of self mappings (T, S) of an intuitionistic fuzzy metric space $(X, M, N,* \lozenge,)$, is said to satisfy the property (E.A) like property if there exist a sequence $\{xn\}$in X such that $\lim_{n\to\infty} Txn= \lim_{n\to\infty} Sxn=z$ for some $z \in T(X)$ or $z \in T(X)$ i.e $z \in T(X) \cap S(X)$

Definition 2.10: Two pairs (A,S) and (B,T) of self mappings of an intuitionistic fuzzy metric space $(X,M,N,*, \lozenge)$, are said to satisfy the common (E.A) property if there exist two sequences $\{xn\}$ and$\{yn\}$ in X such that
$\lim_{n\to\infty} Axn= \lim_{n\to\infty} Sxn= \lim_{n\to\infty} Byn= \lim_{n\to\infty} Tyn=z$ in X for some z in X.

Definition 2.11: Two pairs (A, S) and (B, T) of self mappings of an intuitionistic fuzzy metric space $(X, M, N,*, \lozenge)$, are said to satisfy the common (E.A) like property if there exist two sequences $\{xn\}$ and$\{yn\}$ in X such that
$\lim_{n\to\infty} Axn= \lim_{n\to\infty} Sxn= \lim_{n\to\infty} Byn= \lim_{n\to\infty} Tyn=z$ where $z \in T(X) \cap S(X)$ or $z \in A(X) \cap B(X)$

Example 2.12: Consider $(X, M, N,*, \lozenge)$, be an intuitionistic fuzzy metric space with

$$M(x, y,t) = \frac{t}{t + d(x,y)}$$

And

$$N(x, y,t) = \frac{d(x,y)}{t + d(x,y)}$$

Define self mappings A, B, S and T on X as Ax = x/3, Bx = -x=/3, Sx = x, and Tx = -x for all x X. Then with sequences $\{xn\} = \{1/n\}$ and $\{yn\} = \{-1/n\}$ in X, one can easily verify that $\lim_{n\to\infty} Axn= \lim_{n\to\infty} Sxn= \lim_{n\to\infty} Byn= \lim_{n\to\infty} Tyn=0$ Therefore, pairs (A, S) and (B, T) satisfies the common E.A. property.

Example 2.13: Consider $(X, M, N,* \lozenge,)$, be an intuitionistic fuzzy metric space with

$$M(x, y,t) = \frac{t}{t + d(x,y)}$$

And

$$N(x, y,t) = \frac{d(x,y)}{t + d(x,y)}$$

Define self mappings A, B, S and T on X as

Ax =1 ; where x<=1 and Sx= 2x-4; Where x>=2;
Ax= 2-x, Where x>=1 and Sx = 2-x, x<=2
And
Tx =1 ; where x<=1 and Bx= 2x-4; Where x>=1;
Tx= 2-x, Where x>=1 and Bx = 1/, x>=1
for all x ∈ X. Then with sequences {xn} = {1+1/n} and {yn} = {1-1/n} in X, one can easily verify that lim $_{n\to\infty}$ Axn= lim $_{n\to\infty}$ Sxn= lim $_{n\to\infty}$ Byn= lim $_{n\to\infty}$Tyn=1and A(X)=(- ∞,1], B(X)=(0,1], T(X)= (-∞,1], S(X)=X. Therefore, pairs (A, S) and (B, T) satisfies the common E.A. property

Definition 2.14: A pair of self mappings (T,S) of an intuitionistic fuzzy metric space (X, M, N,*, ◊),is said to be weakly compatible if they commute at coincidence points, i.e., if Tu = Su for some u ∈ X, then TSu = STu. Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces using Common (E.A like) Property and Implicit Relation

MAIN RESULTS

Implicit relations play important role in establishing of common fixed point results.
Let M6 be the set of all continuous functions ψ : [0,1] R and ψ : [0,1] →R satisfying the following conditions:
(A) ψ (u, 1, u, 1, 1, u) < 0 for all u ∈ (0, 1),
(B) ψ (u, 1, 1, u, u, 1) < 0 for all u ∈ (0, 1),
(C) ψ (u, u, 1, 1, u, u) < 0 for all u ∈ (0, 1),
(D) ψ (v, 0, v, 0,0, v) > 0 for all v ∈ (0, 1),
(E) ψ (v, 0, 0, v, v, 0) > 0 for all v ∈ (0, 1),
(F) ψ (v, v, 0, 0, v, v) > 0 for all v ∈ (0, 1).

Theorem3.1: Let A, B, S and T be self mappings of an intuitionistic fuzzy metric space (X, M, N,*, ◊), satisfying the following:

(1) the pair (A, S) and (B, T) satisfies the E.A. property, (2) for any x, y ∈ X, andin M^6 and for all t > 0,
ϕ (M(Ax, By, t),M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t)) ≥ 0
ψ(N(Ax, By, t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, t), N(Ty, Ax, t)) ≤ 0
(2) A(X) ⊆ T(X) or B(X) ⊆S(X). (4) the pairs (A, S) and (B, T) are weakly compatible. (5) S(X) and T(X) are closed subspace of X. Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed.

Now we prove the following:

Theorem3.2: Let A, B, S and T be self mappings of an intuitionistic fuzzy metric space (X, M, N,*, ◊), satisfying the following: (1) the pair (A, S) and (B, T) satisfies the E.A. like property, (2) for any x, y ∈ X, ϕ and ψ in M^6 and for all t > 0,
ϕ (M(Ax, By, t), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t),M(Ty, Ax, t)) ≥ 0
ψ (N(Ax, By, t), N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, t),N (Ty, Ax, t)) ≤ 0
(3) A(X) ⊆ T(X) or B(X) ⊆ S(X).
(4) the pairs (A, S) and (B, T) are weakly compatible.
(5) S(X) and T(X) are closed subspace of X. Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed.

Proof: Since (A, S) and B, T) satisfies the E.A. like property therefore there exist two sequences{xn} and{yn} in X such that
lim $_{n\to\infty}$ Axn= lim $_{n\to\infty}$Sxn= lim $_{n\to\infty}$Byn= lim $_{n\to\infty}$Tyn=z where z ∈T(X) ∩S(X) or z ∈ A(X) ∩ B(X)
Suppose z ∈ T(X) ∩S(X), we have
lim $_{n\to\infty}$ Axn=z ∈ S(X) then z=S(u) for some u ∈ X.
Put x=u and y=yn in (2)
ϕ (M(Au, Byn, t), M(Su, Tyn, t), M(Su, Au, t), M(Tyn, Byn, t), M(Su, Byn ,t), M(Tyn, Au, t)) ≥ 0
ψ (N(Au, Byn, t), N(Su, Tyn, t), N(Su, Au, t),N(Tyn, Byn, t), N(Su, Byn, t),N(Tyn, Au, t)) ≤ 0
As n→∞, we have
ϕ (M(Au, z, t), M(Su, z, t), M(Su, Au, t), M(z, z, t), M(Su, z ,t), M(z, Au, t)) ≥ 0
ψ (N(Au, z, t),N(Su, z, t),N(Su, Au, t),N(z, z, t), N(Su, z, t), N(z, Au, t)) ≤ 0
And Su=z

ϕ (M(Au, z, t), M(z, z, t), M(z, Au, t), M(z, z, t), M(z, z , t), M(z, Au, t)) ≥ 0
ψ (N(Au, z, t), N(z, z, t), N(z, Au, t), N(z, z, t), N(z, z, t), N(z, Au, t)) ≤ 0
$\Rightarrow$ ϕ (M(Au, z, t), 1, M(z, Au, t), 1, 1, M(z, Au, t)) ≥ 0
ψ (N(Au, z, t),0, N(z, Au, t), 0, 0, N(z, Au, t)) ≤ 0
Therefore Au=z=Su. Which shows that u is coincidence point of the pair (A, S). The weak compatibility of A and S implies that ASu=SAu and then
AAu=ASu=SAu=SSu……………………….(1)
Since T(X) is also a closed subset of X, therefore lim $_{n\to\infty}$Tyn=z in T(X) and hence there exists v $\in$ X such that Tv = z = Au = Su. Now, we show that Bv = z.
Take x = u, y = v, we have
ϕ (M(Au, Bv, t), M(Su, Tv, t), M(Su, Au, t), M(Tv, Bv, t), M(Su, Bv, t), M(Tv, Au, t)) ≥ 0
ψ (N(Au, Bv, t), N(Su, Tv, t), N(Su, Au, t), N(Tv, Bv, t), N(Su, Bv, t), N(Tv, Au, t)) ≤ 0
$\Rightarrow$ (M(z, Bv, t), M(z, z, t), M(z, z, t), M(z, Bv, t), M(z, Bv, t), M(z, z, t)) ≥ 0
ψ (N(z, Bv, t), N(z, z, t), N(z, z, t), N(z, Bv, t), N(z, Bv, t), N(z, z, t)) ≤ 0
$\Rightarrow$ ϕ (M(z, Bv, t),1,1, M(z, Bv, t), M(z, Bv, t), 1) ≥ 0
ψ (N(z, Bv, t), 0, 0, N(z, Bv, t), N(z, Bv, t),0) ≤ 0
Therefore, Bv = z = Tv which shows that v is a coincidence point of the pair (B; T). The weak compatibility of B and T implies that BTv=TBv and then
BBv=BTv=TBv=TTv (2) From (1) and (2), we have Az=ASu=SAu=Sz, Bz=BTv=TBv=Tz (3)
Take x = z, y = v, we have
ϕ (M(Az, Bv, t),M(Sz, Tv, t), M(Sz, Az, t), M(Tv, Bv, t), M(Sz, Bv, t), M(Tv, Az, t)) ≥ 0
ψ (N(Az, Bv, t), N(Sz, Tv, t), N(Sz, Az, t), N(Tv, Bv, t), N(Sz, Bv, t), N(Tv, Az, t)) ≥ 0
$\Rightarrow$ ϕ (M(Az, z, t), M(Az, z, t), M(Az, Az, t),M(z, z, t), M(Az, z, t), M(z, Az, t)) ≥ 0
ψ (N(Az, z, t), N(Az, z, t), N(Az, Az, t), N(z, z, t), N(Az, z, t), N(z, Az, t)) ≤ 0
$\Rightarrow$ ϕ (M(Az, z, t), M(Az, z, t), 1, 1, M(Az, z, t), M(z, Az, t)) ≥ 0
ψ (N(Az, z, t), N(Az, z, t), 0, 0, N(Az, z, t), N(z, Az, t)) ≤ 0
Therefore, Az=z=Sz.
Similarly, Bz = Tz = z. Hence, Az = Bz = Sz = Tz=z, and z is common fixed point of A, B, S and T.
Uniqueness: Let z and w be two common fixed points of A, B, S and T. Take x=z and y=w in (2)
ϕ (M(Az, Bw, t), M(Sz,Tw, t), M(Sz, Az, t), M(Tw, Bw, t), M(Sz, Bw,t), M(Tw, Az, t)) ≥ 0
ψ (N(Az, Bw, t), N(Sz,Tw, t), N(Sz, Az, t), N(Tw, Bw, t), N(Sz, Bw, t), N(Tw, Az, t)) ≤ 0
$\Rightarrow$ ϕ (M(z, w, t), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t), M(w, z, t)) ≥ 0
ψ (N(z, w, t), N(z, w, t), N(z, z, t), N(w, w, t), N(z, w, t), N(w, z, t)) ≤ 0
$\Rightarrow$ ϕ (M(z, w, t), M(z, w, t), 1, 1, M(z, w, t), M(w, z, t)) ≥ 0
ψ (N(z, w, t), N(z, w, t), 0, 0, N(z, w, t), N(w, z, t)) ≤ 0
$\Rightarrow$ z=w. hence there is unique fixed point.

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